The Numbers Game

Exercise 1 The 3-number Challenge

Write each positive integer as a combination of the digits 1, 2, and 3: each used at most once, combined via the operations of addition and multiplication only, as well as grouping symbols. For example, 5 can be expressed as 1 \times 2 + 3. You may use parentheses or not, at your own discretion (as long as your expressions evaluate to the given number, following the order of operations). Digits may not be juxtaposed to represent larger whole numbers, so you cannot use the numerals 1 and 2 to create the number 12 for instance.

Exercise 2

Using only 1, 2 and 3, and the rules of the 3-number challenge, show how we could use only addition or only multiplication to arrive at 6.

Exercise 3

Using the numbers 1, 2, 3, 4 only once and the operations + or \times as many times as you like, write an expression that evaluates to 16.

What mathematical property have you had to use to get the values you wanted?

Why does the use of parentheses affect your answer?

Exercise 4

Roma says that collecting like terms can be seen as an application of the distributive property. Is writing 2x + 3x = 5x an application of the distributive property? Show why or why not.

How is x + x = 2x also an application of the distributive property?
Exercise 5

Leela is convinced that \((a + b)^2 = a^2 + b^2\). Do you think she is right? Use a picture to illustrate your reasoning.

Exercise 6

Draw a picture to represent the expression \((a + b + 1) \times (b + 1)\).

Exercise 7

Draw a picture to represent the expression \((a + b) \times (c + d) \times (e + f + g)\).

A Key Belief of Arithmetic

The Distributive Property: If \(a, b, \) and \(c\) are real numbers, then \(a(b + c) = ab + ac\).
Problem Set

The 4-number Challenge

Using the numbers 1, 2, 3, 4 only once and the operations + or × as many times as you like, write an expression that evaluates to all the positive integers between 1 and 25. Several have been done for you.

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<th>Value of expression</th>
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<tbody>
<tr>
<td>1</td>
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<td>14</td>
<td>4 × 3 + 2</td>
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<td>2</td>
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<td>15</td>
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<tr>
<td>3</td>
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<td>16</td>
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<tr>
<td>4</td>
<td>1 + 3</td>
<td>17</td>
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<td>6</td>
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<td>19</td>
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<tr>
<td>7</td>
<td></td>
<td>20</td>
<td>(2+3) × 4</td>
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<td>8</td>
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1. Insert parentheses to make each statement true.
   a. \(2 + 3 \times 4^2 + 1 = 81\)
   b. \(2 + 3 \times 4^2 + 1 = 85\)
   c. \(2 + 3 \times 4^2 + 1 = 51\)
   d. \(2 + 3 \times 4^2 + 1 = 53\)

2. Luke wants to play the 4-number game with the numbers 1, 2, 3, and 4 and the operations of addition, multiplication, AND subtraction.
   Leoni responds, “Or we just could play the 4-number game with just the operations of addition and multiplication, but now with the numbers −1, −2, −3, −4, 1, 2, 3, and 4 instead.”
   What observation is Leoni trying to point out to Luke?
3. Consider the expression: \((x+y+3)(y+1)\).
   a. Draw a picture to represent the expression.
   b. Write an equivalent expression by applying the distributive property.

4. Consider the expression: \((x+3)(y+1)(x+2)\).
   a. Draw a picture to represent the expression.
   b. Write an equivalent expression by applying the distributive property.

5. Given that \(a > b\), which of the shaded regions is larger and why?

6. Consider the expressions 851×29 and 849×31. Which would result in a larger product? Use a diagram to demonstrate your result.
Challenge Yourself:

Consider the following diagram.

a. Edna looked at the diagram and then highlighted the four small rectangles shown and concluded:

\[(x + 2a)^2 = x^2 + 4a(x + a).\]

b. Michael, when he saw the picture, highlighted four rectangles and concluded:

\[(x + 2a)^2 = x^2 + 2ax + 2a(x + 2a).\]

Which four rectangles and one square did he highlight?

c. Jill, when she saw the picture, highlighted eight rectangles and squares (not including the square in the middle) to conclude:

\[(x + 2a)^2 = x^2 + 4ax + 4a^2.\]

Which eight rectangles and squares did she highlight?

d. When Fatima saw the picture, she exclaimed: \((x + 2a)^2 = x^2 + 4a(x + 2a) - 4a^2\). She claims she highlighted just four rectangles to conclude this. Identify the four rectangles she highlighted and explain how using them she arrived that the expression \(x^2 + 4a(x + 2a) - 4a^2\).

e. Is each student's technique correct? Explain why or why not.